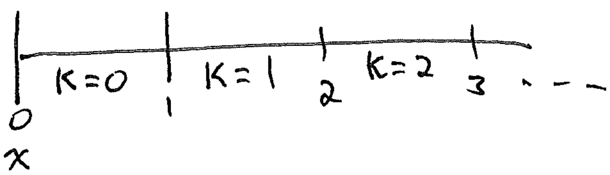
9/3/19

H.W. Answer : $K_x^{(m)} = \frac{1}{m} \cdot \lfloor m \cdot T_x \rfloor$

MISI (Continued)

Expectation of K_x



K_x	Pr
0	$q_x = 1 - p_x$
1	${}_{1|}q_x = p_x - {}_2p_x$
2	${}_{2|}q_x = {}_2p_x - {}_3p_x$
⋮	⋮

$$E[K_x] = 0 \cdot (1-p_x) + 1 \cdot (p_x - {}_2p_x) + 2 \cdot ({}_2p_x - {}_3p_x) + \cdots$$

$$\therefore E[K_x] = p_x + {}_2p_x + {}_3p_x + \cdots$$

Actuarial Notation : $E[K_x] = e_x$

$$\therefore e_x = \sum_{k=1}^{\infty} {}_kp_x$$

Remark: The index starts at $k=1$, not $k=0$

Recall ${}_0p_x = 1$, and so $\sum_{k=0}^{\infty} {}_kp_x = 1 + e_x$

Recursion: (1-year) $e_x = p_x + {}_2p_x + {}_3p_x + \cdots$

$$= p_x + p_x \cdot p_{x+1} + p_x \cdot {}_2p_{x+1} + \cdots$$

$$\therefore \quad e_x = p_x \left(1 + p_{x+1} + {}_2p_{x+1} + \cdots\right)$$

$$\therefore \quad e_x = p_x \cdot \left(1 + e_{x+1}\right) \rightarrow$$ 1-year recursion for e_x

(2-year recursion)

$$e_x = p_x + {}_2p_x + {}_2p_x \cdot p_{x+2} + {}_2p_x \cdot {}_2p_{x+2} + \cdots$$

$$= p_x + {}_2p_x \cdot \left(1 + p_{x+2} + {}_2p_{x+2} + \cdots\right)$$

$$\therefore \quad e_x = p_x + {}_2p_x \cdot \left(1 + e_{x+2}\right)$$

Expectation of T_x

"e-circle sub x"

Actuarial Notation: $\mathring{e}_x = E[T_x]$

$$\therefore \quad \mathring{e}_x = \int_0^\infty t \cdot f_x(t)\, dt$$

other choices

IBP: $u = t$ $\quad v = -{}_tp_x$

$du = dt$ $\quad dv = f_x(t)\, dt$

$uv - \int v\, du$

$$= -t \cdot {}_tp_x \Big|_0^\infty + \int_0^\infty {}_tp_x\, dt$$

<u>Fact</u>: For any $n > 0$, as $t \rightarrow \infty$, then ${}_tp_x \rightarrow 0$ faster than $t^n \rightarrow \infty$.

$$\therefore \quad t \cdot {}_tp_x \xrightarrow{t \rightarrow \infty} 0$$

<u>Punchline</u>: $\mathring{e}_x = \int_0^\infty {}_tp_x\, dt$

Remarks:

1) $$e_x = \sum_{k=1}^{\infty} {}_kp_x$$

$$\mathring{e}_x = \int_0^{\infty} {}_tp_x \, dt$$

2) $$e_x < \mathring{e}_x < 1 + e_x$$

Test 1 Review

H.W. Exercises 1-18 from MISI

#16)